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Software for Flow Distribution in Electronic Rack Structures

by

R. J. Pieper

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Brief Description

A FORTRAN CODE has been developed which predicts pressures and flow rates in a piping network for the regimes of laminar, transition, and turbulent flow. A novel and effective scheme for modeling the transition region between laminar and turbulent flow is demonstrated. A proposed linear indicator for the deviation from laminar behavior is introduced into the analysis in order to facilitate the computational task. Although the turbulent flow problem is inherently nonlinear, the method of analysis described herein, typically employed with linear electrical networks, is found to converge rapidly to an approximate solution without user-specified *a priori* guesses.

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1. Introduction

An interest in the properties of incompressible fluid flow in pipes can undoubtedly be traced as far back as the era of the Roman empire. A brief synopsis of the historical development of fluid mechanics can be found in many of the introductory level texts [Vennard (1961) for example].

The problem of accurately solving for the parameters in incompressible fluid flow in piping networks is dependent upon the single-pipe model for fluid flow. Many semi-empirical methods for characterizing the flow properties have appeared in the literature [Matthew (1981), Wuori (1993), and Churchill (1977)]. Since the early work by Feigenbaum (1978), see for example [Hofstadter (1981)], on chaos in simple systems, considerable progress has been made on analytically characterizing turbulence in fluids [Landau and Lifshitz (1987)]. Nevertheless, many engineering problems are solved using the Stanton diagram, which shows Nikuradse's measured data.

For engineering problems which require repetitive numerical evaluation, an empirical relationship facilitates the task and various investigators [Tsang and Kee (1987), Round (1980, 1985), and Colebrook (1939)] have proposed such formulae. These formulae have the drawback that the transition region between laminar and turbulent flow is inadequately represented. Only the fluid flow properties in the laminar region are fairly well understood [Vennard (1961)] using methods of fluid analysis.

One of the earliest and most well known methods for solving pipeline networks is attributed to Hardy Cross [Giles (1962) and Lindeburg (1982)]. The Hardy Cross scheme, not only requires an *a priori* guess on the initial flows, but upon comparison with more recently reported methods [Carnahan and Christensen (1972), Bending and Hutchinson (1973), Gay and Preece (1975, 1977)], it is found to be less efficient. Finally, in a recent Naval Postgraduate School (NPS) thesis [Ellis (1993)], a pipeline

method formalism based on the Cholesky method for matrix reduction has been investigated. Although the mathematical analysis of the scheme is fairly efficient, the physical modeling did not encompass conditions for other than laminar flows.

The main purpose of this report is to extend the range of flow to include transition and turbulent flow regimes. As this is primarily a feasibility study, various facilitating assumptions were made. First, it is assumed that English units are preferred for input/output. Second, all pipes are assumed to have uniform circular cross-sections.

2. Physical Modeling

For incompressible fluids, two of Bernoulli's rules [Lindeburg (1982)] for conservation of mass and energy are, the continuity equation

$$Q = A_1 V_1 = A_2 V_2 \tag{1}$$

and Bernoulli's law

$$\frac{\dot{w}_s}{Q} = \rho \frac{(V_2^2 - V_1^2)}{2} + (p_2 - p_1) + \rho g (z_2 - z_1) + \rho g h' \tag{2}$$

where the subscript 2/1 refers to positions 2/1 and the symbol lists given in Section 10 defines the other relevant parameters. The head loss, as defined by the Darcy-Weisbach equation, is given by

$$h' = \frac{f L V^2}{2 g d} \tag{3}$$

where L is the pipe length, d is the hydraulic diameter, and f is the friction factor. For circular cross-section pipes the hydraulic diameter and the physical diameter are identical. Using similitude analysis [Vennard (1961)], it is possible to show that the friction factor depends only on two dimensionless parameters, the Reynolds number

$$\mathrm{Re} = \frac{\rho V d}{\mu} \tag{4}$$

where μ is the dynamic viscosity, and the relative roughness

$$\epsilon = \frac{e}{d} \tag{5}$$

where e is the roughness of the pipe.

A sourceless section of pipe which is horizontal ($z_2 = z_1$), and with constant cross-section so that $V_2 = V_1$, will, from eq (2) and eq (3), then have a pressure drop

$$p_1 - p_2 \equiv h \equiv \rho g h' = \frac{\rho f L V^2}{2d} \tag{6}$$

or after use of eq (1)

$$h = f \frac{L \rho V}{2dA} Q \tag{7}$$

This suggests the useful electrical analogy with Ohm's law where pressure drop corresponds to voltage and fluid flow corresponds to current. It follows that consistent with the analogy, the effective resistance of the pipe is then given by

$$R = f \frac{L \rho V}{2dA} \tag{8}$$

where the units of this resistance are defined by the ratio of pressure to flow.

It is observed that the problem of predicting fluid flow in pipes now reduces to accurate characterization of the friction factor. For the laminar flow domain the analytically derivable [Vennard (1961)] result

$$f = \frac{64}{\mathrm{Re}} \tag{9}$$

agrees very well with measurement out to $\mathrm{Re} \leq 2100$ which is the well known limit in pipes, ducts, tubes, and channels for laminar flow. After substitution of eq (9) into eq (8) it is found that:

$$R_\ell = \frac{32 L \mu}{d^2 A} \tag{10}$$

and the corresponding relationship for head loss is

$$h_\ell = \frac{32L\mu}{d^2A}Q \tag{11}$$

This is in agreement with an expression cited by Bending and Hutchison (1993). The subscript ℓ has been used to indicate the laminar flow domain. Note from eq (10) that, as expected, the pipe resistance is independent of the flow rate in the laminar regime.

In order to easily characterize the degree to which the fluid flow is turbulent, the head loss can be expressed as

$$h = h_\ell \times \psi \tag{12}$$

where the turbulence factor, denoted ψ, is a linear gauge of the deviation of the system from laminar flow. After noting that eq (7) can be reexpressed as

$$h = \frac{32L\mu}{d^2A}Q \times \frac{f\rho V d}{64\mu} \tag{13}$$

it can be concluded from eq (4) that, in agreement with the definition of the Reynolds number, the turbulence factors satisfy

$$\psi = f\frac{\mathrm{Re}}{64} \tag{14}$$

As seen from eq (9) $\psi = 1$ for laminar flow. In the next section it will be shown that for transition and turbulent flow $\psi > 1$.

The pipe resistance can now be expressed as

$$R = R_\ell \times \psi \tag{15}$$

where as previously noted, R_ℓ does not depend on the flow rate.

3. Friction Factor Estimation for Nonlaminar Fluid Flow

For purposes of the development here it is necessary to introduce two distinct non-laminar regions. First, the transition region satisfying

$$\mathrm{Re}_o \geq \mathrm{Re} \geq 2100 \tag{16}$$

where Re_o is usually cited typically between 3500 and 4000 [Vennard (1961)] and for the turbulent region

$$\mathrm{Re} \geq \mathrm{Re}_o \tag{17}$$

One of the most commonly employed empirical formulas for the turbulent flow domain defined by eq (17) is attributed to Colebrook (1939)

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{e}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \tag{18}$$

Although, as seen in eq (18), the dependence of the friction factor on Reynolds number is implicit, the expression will converge rapidly upon iterative substitution.

To date the most viable predictors for the transition region, see eq (16), are empirical. The Stanton diagram [Vennard (1961)] shows smoothly varying curves which are based on measurements in the transition region. The Moody diagram, which only plots expressions for eq (9), for $\mathrm{Re} < 2100$, and eq (18), for $\mathrm{Re} > \mathrm{Re}_o$, is not useful for the transition region. Bending and Hutchinson (1973) suggested, in a computer analysis of piping networks, that a linear interpolation between the laminar and the turbulent domain be made to provide friction data in the transition region. For the network analysis scheme to be described herein, a linear approximation for the transition region was found to be insufficiently accurate upon numerical testing. In particular, the iterative program with the linear approximation would either not

converge or converge very slowly. The problem in this scheme was the dramatic discontinuity in the first derivative at the edges of the transition region. A greatly improved modeling scheme, in terms of convergence, is given by

$$f = \frac{\text{Re}}{2100} + \left[f(\text{Re}_o) - \frac{\text{Re}}{2100}\right] \sin\left[\frac{\pi}{2}\frac{(\text{Re} - 2100)}{(\text{Re}_o - 2100)}\right] \tag{19}$$

where $f(\text{Re}_o)$ was calculated via the Colebrook model eq (18). Note that, like the linear approximation model, continuity is preserved at the edges of the transition region. Because, as seen from eq (19), the slope at the transition edges is zero, the discontinuity in the slope is much less dramatic and the convergence was found to be much more rapid. Figure 1 is a graphical representation for these curves in standard log-log format for various relative roughness factors.

It is worth noting that the work by Churchill (1977) agreed very well with the scheme just described for Re > 2100 and provided an explicit formula for the friction factor, f. However, for Re < 2100 (laminar flow) the expression deviated *significantly* from the classical expression of eq (9). Hence it was not used.

The argument supporting the claim that the turbulence factor eq (14) is generally greater than unity is loosely based on the following observations. First, in 1913 Blasius proposed an empirical relationship for smooth pipe, i.e., $\epsilon = 0$ in the turbulent flow regime [Vennard (1961)]

$$f_{\text{smooth pipe}} = \frac{0.316}{\text{Re}^{1/4}} \qquad 3000 < \text{Re} < 100{,}000 \tag{20}$$

Second, the construction algorithm for predicting f in the transition region, eq (19) requires that

$$f_{\text{transition}} \geq f_{\text{laminar}} = \frac{64}{\text{Re}} \tag{21}$$

This leads to the inequality that is established from a comparison of eq (20), eq (21),

and Fig 1

$$f_{\text{rough pipe}} \geq f_{\text{smooth pipe}} \geq f_{\text{laminar}} = \frac{64}{\text{Re}} \tag{22}$$

and is supported by analysis and measurement [Vennard (1961)]. The inequality, $\psi \geq 1$, then follows directly from the definition of the turbulence factor, ψ, given by eq (14).

The turbulence factor has been introduced into this report for two reasons. Although the friction factor is a linear indicator for the system response of the fluid dynamics, it is not a linear indicator for the deviation of the system from laminar behavior. Note that the quantity (Re − 2100) is a *nonlinear* indicator for the deviation from laminar behavior. The purpose of this work has been to extend the Ellis (1993) effort to encompass nonlaminar regimes. The turbulence factor will readily demonstrate the need for this extension. Second, it turns out that the laminar pipe resistance, R_ℓ, can be computed at the onset of the problem and only the turbulence factors need to be updated in the iterative calculations of eq (15) rather than to compute eq (8). In brief, the turbulence factors provide some computational advantages which, for large piping networks, would be significant.

4. Basic Circuit Modeling

Following the electrical analogy for an arbitrary k^{th} pipe, a pressure difference, Δp_k, across a pressure source, Δp_{sk}, in series with a pipe resistance, r_k, with a flow q_k, satisfies

$$\Delta p_k = q_k r_k - \Delta p_{sk} \tag{23}$$

consistent with the convention established by Fig 2. The corresponding relation in terms of a current source is given by

$$q_k = q_{sk} + y_k \Delta p_k \tag{24}$$

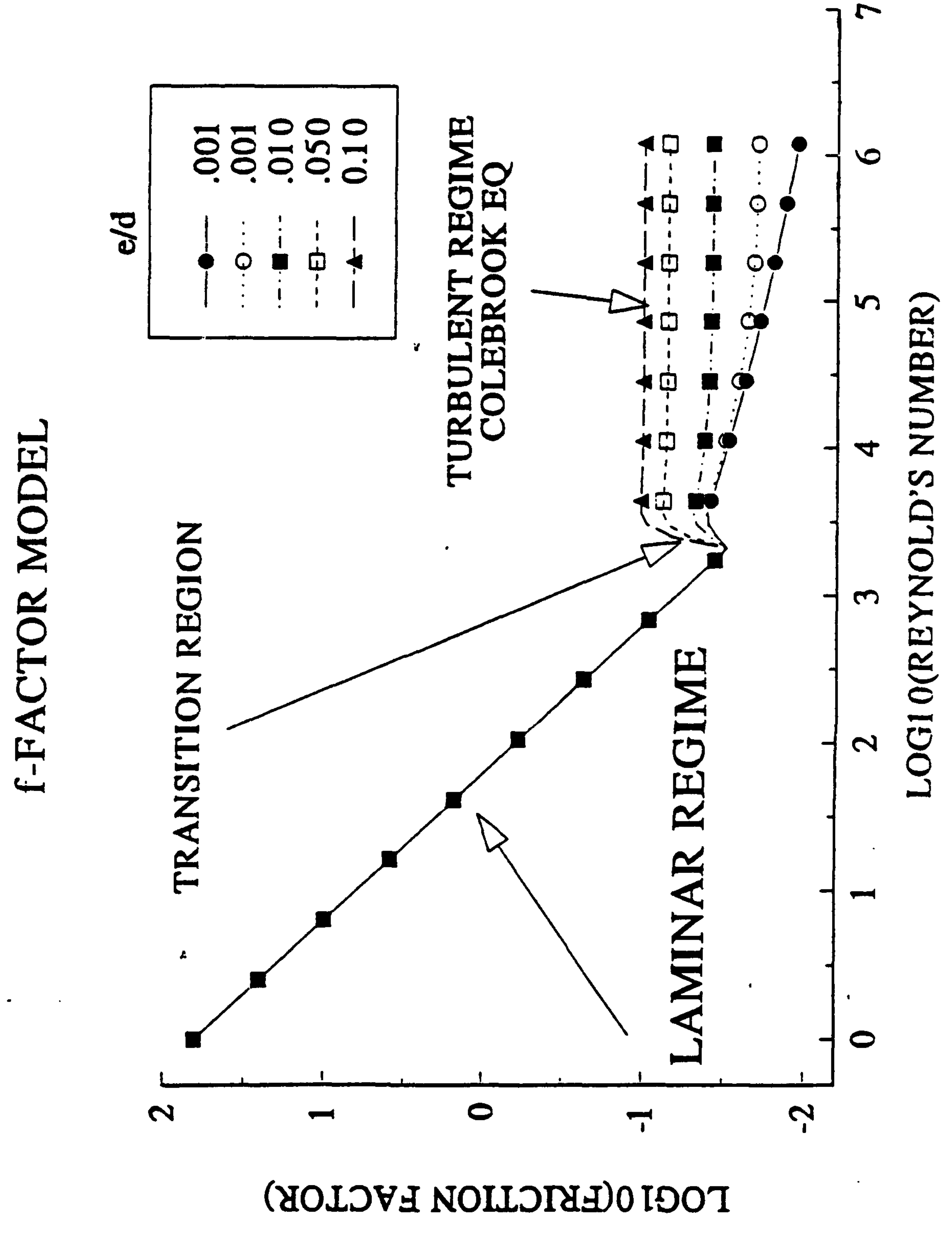


Figure 1: Friction factor model.

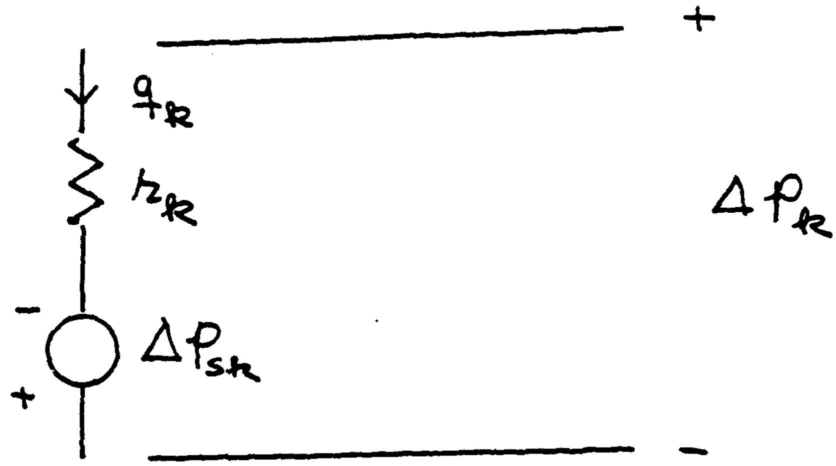


Figure 2: Pressure source circuit model.

where $y_k = 1/r_k$. Figures 2 and 3 define the nomenclature and the topology. Equation (24) can be compactly represented for all branches or pipes in matrix format

$$\mathbf{Q} = \mathbf{Q}_s + \mathbf{Y}\Delta\mathbf{P} \tag{25}$$

where it is seen that eq (24) and eq (25) have unresolved unknowns associated with flow rate and pressure. The condition of continuity at the nodes

$$\sum_k q_{kj} = 0 \qquad j = 1, 2, \ldots n_T \tag{26}$$

where n_T is the total number of nodes in the network, provides enough information to resolve the flow rates and pressure drops. A formulation for this process is presented in the next section.

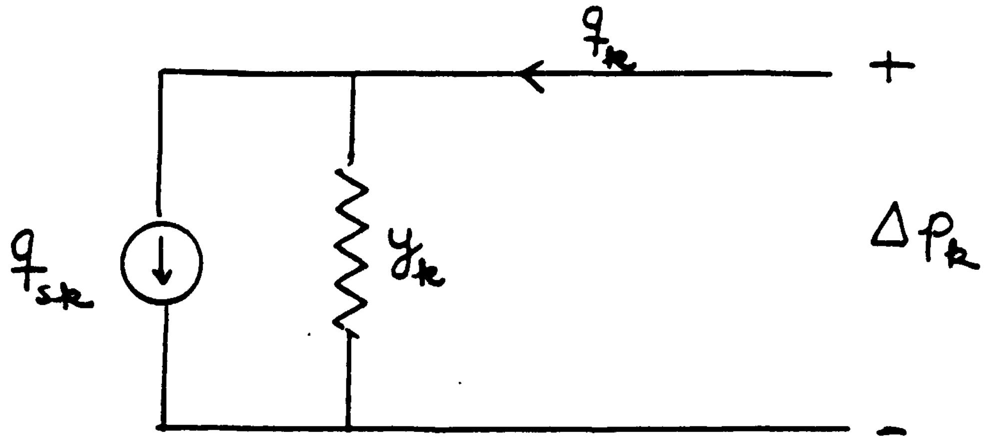
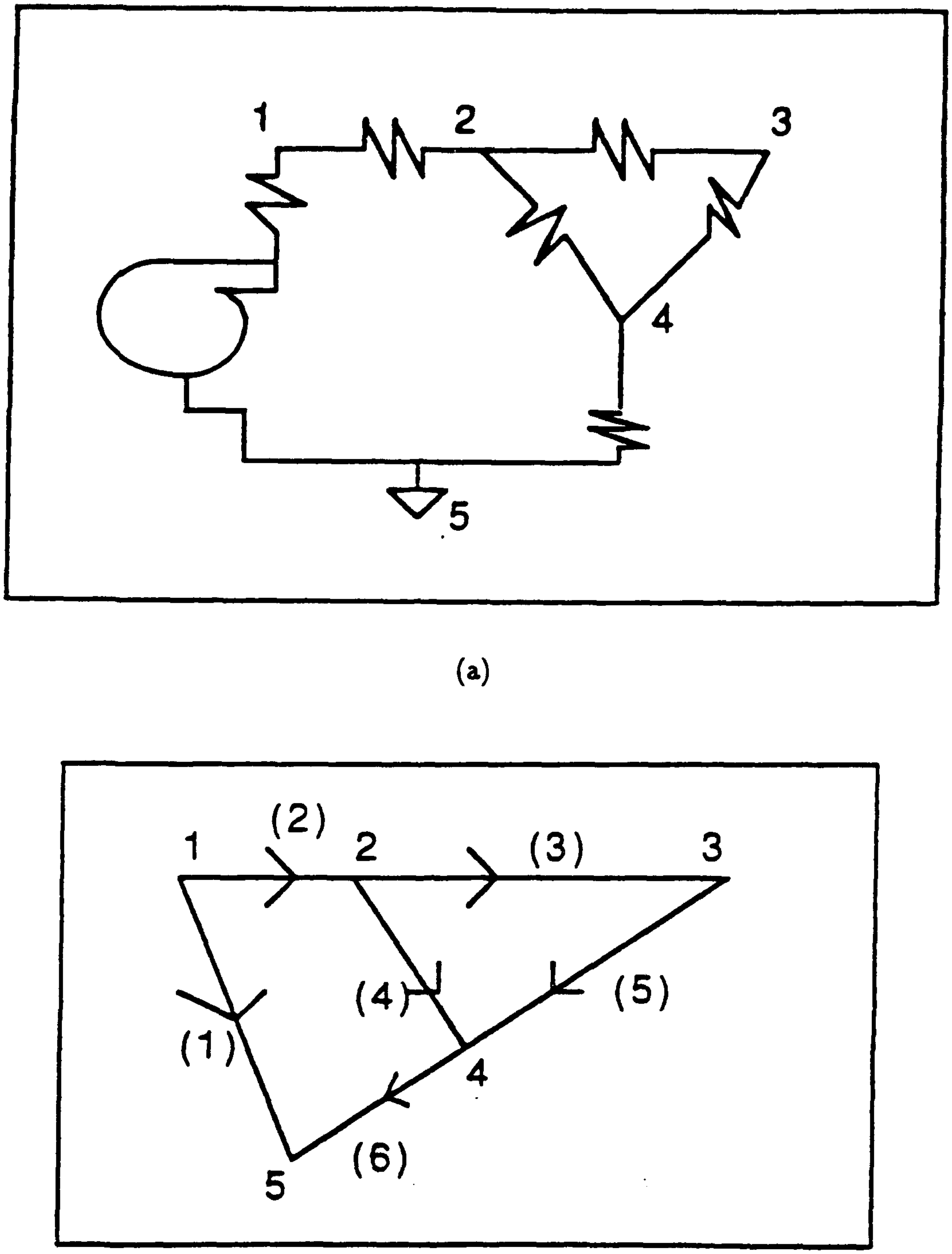


Figure 3: Flow source circuit model.

5. Mathematical Formulation

The mathematical formalism is presented via an illustration taken from the Ellis (1993) thesis. The piping network shown in Fig 4a has five nodes and six connecting branches. It is also seen that the branch between nodes 1 and 5 contains a pump. This pump provides a source for fluid flow at its high pressure side shown at node-1 with an arrow. The low pressure side is the corresponding sink. The corresponding flow graph is shown in Fig 4b. Each branch now has an *arbitrarily chosen* orientation arrow which defines the convention for positive fluid flow. Note that consistent with the flow direction shown for branch 1, the source shown on Fig 4a would be assigned a negative numerical vale. The first step in the mathematical formalism is to create both a network representation for the fluid pipeline network and a corresponding flow graph. The flow rates in the b branches, in this case 6, can be represented as a $b \times 1$



(a)

(b)

Figure 4: a) Six branch pipeline network; b) Associated flow graph for a) [15].

column vector

$$\mathbf{Q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \vdots \\ q_B \end{bmatrix} \tag{27}$$

The elements of the augmented node-branch incidence matrix, c_{ij}^a, can be def
as

$$c_{ij}^a = 1 \quad \text{Branch } j \text{ leaves node } i \tag{28a}$$

$$c_{ij}^a = -1 \quad \text{Branch } j \text{ enters node } i \tag{28b}$$

$$c_{ij}^a = 0 \quad \text{otherwise} \tag{28c}$$

where $i = 1, 2, \ldots, n_T$ nodes and $j = 1, 2, \ldots, b$ branches. This leads to:

$$\mathbf{C}^a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \\ -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{29}$$

where, in general, $\mathbf{C}^a$ has b columns and n_T rows. It is apparent that for each column, which is associated with a node,

$$\sum_k c_{jk}^a q_k = 0 \tag{30}$$

or more formally

$$\mathbf{C}^a \mathbf{Q} = 0 \tag{31}$$

consistent with flow rate conservation at all nodes (see eq (26)). In order to reduce the order of the problem by 1 it is useful to define one of the nodes as the ground or datum reference. In principle it should not matter which node is taken as this datum

node, however, in the coded implementation it is assumed that the circuit nodes are numbered such that the datum node is the node bearing the highest numerical designator. For example, as seen on Figs 4a, b, the node-5 is the datum node. Because the assignment of nodes is independent of the assignment of branches and is arbitrary, there is no sacrifice in the generality of the method. Following this point, the node-branch incidence matrix, **C**, can be defined according to the rules dictated by eq (27) with the index i satisfies the condition $i = 1, 2, \ldots, n$ where

$$n = n_T - 1 \tag{32}$$

Thus, for this example, the node-branch incidence matrix is

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \tag{33}$$

In general **C** has b columns and n rows. It follows from eq (30) that

$$\mathbf{C}\,\mathbf{Q} = 0 \tag{34}$$

which forces a condition of flow rate conservation or continuity at all nodes.

The node pressures, defined with respect to the reference datum node, can be represented by the column vector

$$\mathbf{P} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \tag{35}$$

which, for the example under consideration, the node pressure vector **P** is a 4×1 column vector since $n = 4$. The node branch incidence matrix can be used to predict the head losses from the rule

$$\mathbf{H} = \mathbf{C}^T \mathbf{P} \tag{36}$$

where the branch head loss vector, represented as:

$$\mathbf{H} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_B \end{bmatrix} \tag{37}$$

can be calculated using eqs (11) and (12). For the example being examined, $\mathbf{H}$ is a 6×1 column vector, corresponding to the number of branches.

In Section 4 the pressure drop vector was introduced without any assumptions regarding the physical modeling introduced in Section 2. Under the practical assumptions made leading to eq (6), it is correct to let

$$\mathbf{\Delta P} \longrightarrow \mathbf{H} \tag{38}$$

in eq (25), which leads to

$$\mathbf{Q} = \mathbf{Q}_s + \mathbf{Y}\,\mathbf{H} \tag{39}$$

Premultiplication of this equation by $\mathbf{C}$ leads to:

$$\mathbf{O} = \mathbf{C}\,\mathbf{Q}_s + \mathbf{C}\,\mathbf{Y}\,\mathbf{H} \tag{40}$$

where the left hand side of eq (40) is predicted from eq (34). After substitution of eq (36) into eq (40) and solving for $\mathbf{P}$, it follows that

$$\mathbf{P} = \mathbf{Y}_n^{-1}\tilde{\mathbf{Q}} \tag{41}$$

where

$$\tilde{\mathbf{Q}} \equiv -\mathbf{C}\,\mathbf{Q}_s \tag{42a}$$

and

$$\mathbf{Y}_n \equiv \mathbf{C}\mathbf{Y}\mathbf{C}^T \tag{42b}$$

Given the matrix $\mathbf{Y}_n$ and the source vector $\tilde{\mathbf{Q}}_s$, any number of matrix inverting schemes [Hamming (1962)] could be employed to evaluate eq (40). As suggested in the Ellis (1993) thesis the very efficient Cholesky reduction algorithm is applicable here because the matrix $\mathbf{Y}_n$ is, not only symmetric, but positive definite. The Cholesky reduction into upper and lower triangular matrices permits a rapid matrix inversion using Gaussian elimination as described in Appendix A.

Once the node pressures given by eq (40) are obtained the branch head losses and flow rates are easily calculated from eq (36) and eq (39), respectively. Note that, once the flow rate vector is obtained, the flow rate velocities can be computed according to the rule

$$\mathbf{V} \equiv \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_B \end{bmatrix} = \frac{1}{A}\mathbf{Q} \tag{43}$$

in agreement with eq (1). The corresponding velocity magnitudes, $(|v_1|, |v_2|, \ldots, |v_B|)$ can then be applied with eq (4) to calculate the Reynolds numbers, the friction factors, and the turbulence factors using eq (14). The pipe resistances are calculated from eq (15) and the corresponding admittances, $y_1, y_2, \ldots, y_b$, follow from

$$Y = \begin{bmatrix} 1/r_1 & 0 & 0 & \cdots & 0 \\ 0 & 1/r_2 & & & 0 \\ 0 & & \ddots & & 0 \\ \vdots & & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1/r_b \end{bmatrix} \tag{44}$$

If the current turbulence factors differ significantly from the previous turbulence factors according to the rule

$$\text{MAXERR} \geq \max|\psi_k(\text{previous}) - \psi_k(\text{current})| \; : \quad \{\text{branches } k = 1, b\} \tag{45}$$

the process is repeated. The Fortran input parameter MAXERR defines the maximum deviation in the turbulence factors between iterations. It is assumed initially, in the

first step of the iterative scheme, that the flow is in the laminar regime, that is, all turbulence factors = 1.

6. Input/Output: Parameters, Conventions, Procedures

It follows for the definition of specific weight that

$$\gamma = \rho g \tag{46}$$

where g, the gravitational constant, is 32 ft/sec^2. Using eq (46) the computer program computes the density which is needed for calculation of the Reynolds numbers eq (8).

Figure 5 defines the convention for distinguishing the entry (F) and exit (T) nodes of a particular branch. The arrow defines the direction of positive flow.

Figure 5: Convention defining nodes.

The required inputs to the FORTRAN program code `flow.for` provided in Appendix A are displayed in Table 1.

The calculations are done in formal English units of (ft, Lb, sec). However, the inputs and outputs are, when appropriate, given in conventional units. Conversion factors for these cases are shown in Table 2.

Table 1: Input Variables

Symbol	Section	Units	
R_{eo}	§3	—	typical 3500
MAXERR	§5	—	typical 0.05
HLCF	§2	—	typical 60.0
n_T	§5	—	depends on network
b	§5	—	depends on network
μ	§2	10^{-5} lb-sec/ft^3	typical H_2O, 3.75
W	§6	lb/ft^3	typical H_2O, 64.0
node specification $-F$, $-T$	§6	—	depends on pipe
L	§2	ft	depends on pipe
d	§2	inches	depends on pipe
e	§2	mil	depends on pipe
source specification Q_s	§5	gal/min	depends on pump(s)

Table 2: Conversion Factors

Description	Symbol	Formal Units	Conventional Units	Conversion Factor
Flow rate	Q	ft^3/sec	gal/min	450
Pressure	P	lb/ft^2	psi	1/144
Roughness	e	ft	mil	12,000

In order to facilitate the process of batch input, the code has been designed to read the input file `flowinp.dat` rather than require the user to laboriously submit the data in the interactive mode. In this way a maximum amount of flexibility can be incorporated into the program without creating a tedious data entry procedure for the user of the code. Thus, the "user friendly" goal has been kept in mind.

The outputs from the code `flow.for`, which are listed by branch number (BR), are displayed in Table 3.

Where appropriate both text symbol and fortran symbol have been specified in Table 4. For example, PF and PT refer to node pressures at entry and exit, respectively. In some practical cases it may be desirable to employ the equivalent of an "ideal" constant flow rate pump, that is, $q_k = q_{sk}$ in eq (24). This is readily handled by forcing $y_k \to 0$ or equivalently $r_k \to \infty$. Because of the head loss coupling factor rule-of-thumb introduced at the end of Section 3, the most convenient way to force the pump source to behave ideally is to let $d_k \to 0$. In practice, because of numerical instabilities, if $y_k = 0$, it is recommended that the user set

$$d_k = 10^{-3} \times \min\{d_i\} \qquad i = 1, 2, \ldots, b \qquad \text{but } i \neq k\,. \tag{47}$$

A hardcopy of the input/output for the piping networks tested is provided in Appendix B. A more detailed description of these cases is covered in the next section.

Table 3: Output Variable

Symbol	Section	Units
BR	§5	—
Q	§5	gal/min
PF	§6	10^{-3} psi (mpsi)
PT	§6	10^{-3} psi (mpsi)
HEAD = H	§5	10^{-3} psi (mpsi)
ERR	see RHS eq (44)	—
Re	§2	—
TBF = ψ	§2	—
Y	§5	(gal/min)/mpsi

7. Examples

General Observations

Several general observations can be easily confirmed from the data (see Appendix B for all the case examples discussed in this section). Specifically,

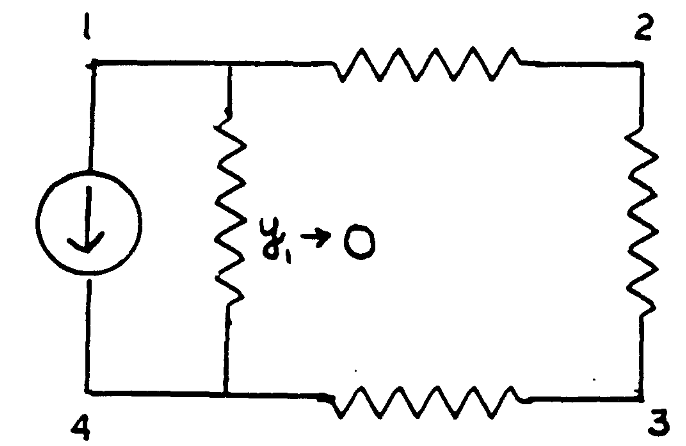
$$\mathbf{PF - PT = HEAD} \qquad (p_F - p_T = h_k)$$

and

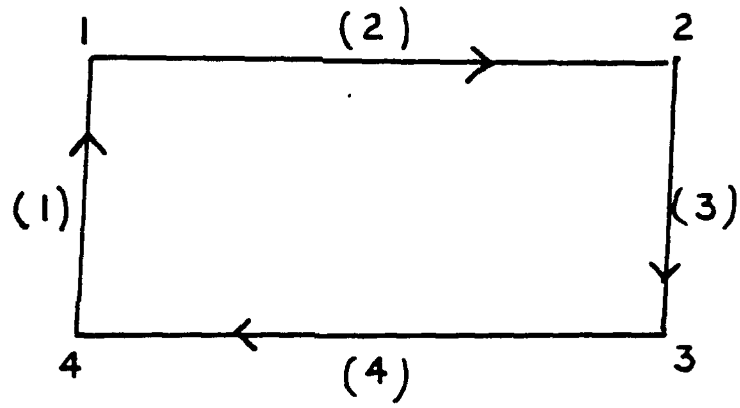
$$\mathbf{Q = Q_s + Y\,HEAD} \qquad (q_k = q_{sk} + y_k h_k)$$

In all the examples the sink node of the pump was arbitrarily taken as the datum node for convenience of interpretation. The node pressures under columns PF and PT are as expected, strictly positive. In all three examples some or all of the pipeline segments were operating in the nonlaminar regime. This is evident by checking the TBF *columns of* each example in Appendix B. The associated Reynolds number is also provided. The branch errors were calculated according to eq (45). It is easily checked that the MAXERR parameter, line 2 of the input set, put a ceiling on these branch error values.

CASE A — A four branch pipeline network with an ideal source (see Fig 6a and 6b). This case illustrates the generation of an ideal flow source. Note, as discussed, this condition can be obtained by making the pipe diameter of the source branch relatively small. The associated Reynolds number and turbulence factors will be artificially high and may not, as represented in the line 1 output for Case A, be within the defined format.



(a)



(b)

Figure 6: a) 4 branch with ideal source; b) 4 branch flow graph for a).

<u>CASE B</u> — A six branch pipeline network with nonideal source (see Fig 4a and 4b). Note, as seen here, the effect of the nonzero admittance of node 1 is to impede the flow of the source into the rest of the pipeline circuit. Also, the source is required to be negative because the pump is physically driving the flow opposite to the convention defined by the arrow of branch #1.

CASE C — A 12 branch piping network for shoebox cooling rack (see Fig 7). Note that several of the branches have negative Q values. The physical interpretation is that these cases have a fluid flow opposite to the convention defined on Figure 7.

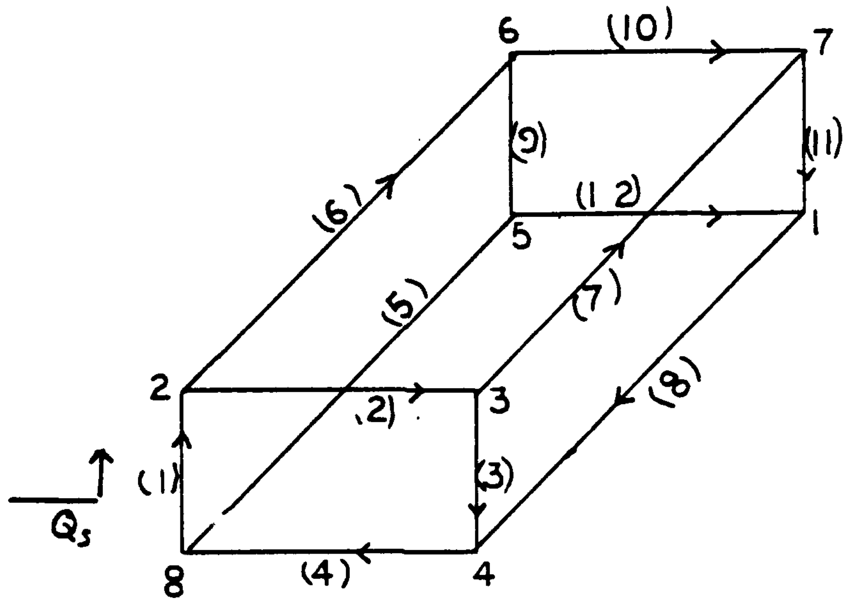


Figure 7: 12 branch shoebox cooling rack.

8. Future Enhancements in the Modeling

Although the work described herein extends the Ellis (1993) thesis by including the nonlaminar flow regime, it still should be considered as preliminary. For example:

- The network formalism discussed herein should be compared with the benchmark Hardy Cross method [Lindeburg (1987)] as well as other more recently proposed schemes [Carnahan and Christensen (1972), Bending and Hutchinson (1973), Gay and Preece (1975, 1977)] in order to assess the relative computational efficiencies.

- The next major modeling development in the algorithm would be the addition of the capability of evaluating the node to node heat transfers.

- Using the theory for compressible fluid flow [Vennard (1961)] required, for example, when the cooling fluid is a gas, the developed scheme could be extended to allow for compressible fluids. Due to the sensitivity of gas properties to temperature this development would have to include effects due to heat transfer.

- An alternate but similar mathematical formalism could be derived for pumps best characterized as constant pressure sources. It is certainly possible to "Theveninize" the source in the context of the present mathematical formalism. However, it should be noted that because of the nonlinearity inherent in the model, this Theveninization would need to be repeated iteratively, decreasing the efficiency of the process.

- As previously noted in Section 3, a significant improvement in the rate of convergence of the method was obtained by reducing the discontinuity in the friction factor derivatives at the transition edges. It should be possible to apply mathematical adjustments on eq (19), for example, using a technique known as Hermite or "osculating" interpolation [Hamming (1962)], in order to completely eliminate the slope discontinuity. The potential gain in the rate of convergence could be significant.

- Various mundane, but no doubt practical embellishments could also be considered. For example, an option of metric system units could be added. Also, an option to specify pipe bends, vertical elevation, and cross-section types could be added. If commercial viability is of concern, the program could be dovetailed with CAD software.

- The required background investigation revealed that despite a long history of developments on modeling fluid flow in pipes, only recently have mathematical

methods involving the theory of chaos begun to unravel and predict the properties of nonlaminar fluid flow. A small-scale physical model which predicts *all* the measured properties on the Stanton diagram is at this date unavailable.

9. Conclusion

A mathematical model for predicting laminar and nonlaminar flows in pipeline networks has been proposed and successfully tested. This scheme, upon iterative application, has been found to converge to an approximate solution. The computational efficiency of this convergence process was found to be highly dependent on the friction factor modeling in the transition region between laminar and turbulent behavior. In particular, a significant improvement in the convergence rate was obtained by replacing the previously proposed linear model with a nonlinear model having a less dramatic first derivative discontinuity. A linear gauge for the deviation from laminar behavior, referred to as the turbulence factor, was introduced in this report in order to facilitate the computational task.

10: Nomenclature

Roman Letter Symbols		Units
A	pipe cross-sectional area	ft^2
d	pipe diameter	ft
e	pipe roughness	ft
f	friction factor	—
g	gravitational acceleration	ft/sec^2
h	head loss due to friction	lb/ft^2
P	pressure	lb/ft^2
Q	flow rate	ft^3/sec
R	pipe resistance	$Lb\text{-}sec/ft^5$
Re	Reynolds number	—
V	velocity	ft/sec
$\dot{w}_s$	time-rate-change of work output (pump)	lb-ft/sec
Y	pipe admittance	$ft^3/(lb\text{-}sec)$
z	elevation	ft

Greek Letter Symbols		
γ	specific weight	lb/ft^3
ϵ	pipe roughness ratio	—
μ	viscosity	$lb\text{-}sec/ft^3$
ρ	density	$slug/ft^3$

Appendix A: Cholesky Reduction

This appendix provides a more complete description on the Cholesky reduction algorithm [Kraus (1992)]. Positive definite, symmetric matrices can be factored according to the rule

$$\mathbf{M} = \mathbf{L}\mathbf{L}^T \tag{A1}$$

where $\mathbf{L}$ is an upper triangular matrix. Because of the applicability of this theorem [Ellis (1993), Kraus (1987)] to the network matrix $\mathbf{Y}_n$ eq (41b) it follows that

$$\mathbf{Y}_n = \mathbf{L}\mathbf{L}^T \tag{A2}$$

and for eq (40)

$$\mathbf{L}\mathbf{L}^T\mathbf{P} = \tilde{\mathbf{Q}} \tag{A3}$$

The advantage of the factorization eq (A1) is now apparent since eq (A3) can be broken into two parts: First,

$$\mathbf{L}\mathbf{J} = \tilde{\mathbf{Q}} \tag{A4a}$$

and second,

$$\mathbf{L}^T\mathbf{P} = \mathbf{J} \tag{A4b}$$

which are easily solved in succession via Gaussian elimination to determine first the column vector $\mathbf{J}$ as an intermediate step then the derived node pressures $\mathbf{P}$.

Appendix B: Input/Output Hardcopy

```
****************     INPUT FORMAT *************************
REYNOLDS' NUMBER FOR RIGHT EDGE OF TRANSITION REGION
ERROR TOLERANCE IN CALCULATION
THE HEADLOSS COUPLING FACTOR (TYPICAL NUMBER 60 )
THE NUMBER OF NODES
THE NUMBER OF BRANCHES
THE VISCOSITY X 10 -5  LB-SEC/FT^3
THE DENSITY LB/FT ^3
STARTING NODE, ENDING NODE, LENGTH(FT),DIAMETER(IN), ROUGHNESS(MILS)
                 NOTE FOR IDEAL CURRENT SOURCE SET DIAM=.001X SMALLEST
THE NUMBER OF SOURCES
SOURCE STRENGTH  GAL/MIN
SOURCE BRANCH
************************************************************
INPUT (CASE A  4 BRANCHES IDEAL SOURCE)
3500.0
0.005
60.0
4
4
3.75
64.
4,1, 1.0,.001,1.0
1,2, 1.0,1.0,2.0
2,3, 1.0,1.0,0.5
3,4, 1.0,1.0,0.6
1
6.0
1


OUTPUT CASE A
BR      Q        PF       PT      HEAD      ERR      Re        TBF      Y
  1    6.00       .0    282.1   -282.1    .0039 ******* *******    .0000
  2    6.00    282.1    183.8     98.2    .0032 10806.2      5.6    .0609
  3    6.00    183.8     92.1     91.7    .0032 10806.2      5.2    .0652
  4    6.00     92.1       .0     92.1    .0032 10806.2      5.3    .0649
***********************************************************************
INPUT (CASE B  6 BRANCH PIPELINE NETWORK)
3500.0
```

```
0.01
60.0
5
6
3.75
64.
1,5, 1.0,1.0,1.0
1,2, 1.0,1.0,2.0
2,3, 1.0,1.0,0.5
2,4, 1.0,1.0,0.6
3,4, 1.0,1.0,0.6
4,5, 1.0,1.0,5.0
1
-6.0
1
```

OUTPUT CASE B

BR	Q	PF	PT	HEAD	ERR	Re	TBF	Y
1	-1.75	25.1	.0	25.1	.0073	3149.7	2.0	.1679
2	1.75	25.1	14.6	10.6	.0075	3149.7	2.1	.1645
3	.58	14.6	12.9	1.7	.0000	1049.9	1.0	.3409
4	1.17	14.6	11.2	3.4	.0000	2099.8	1.0	.3409
5	.58	12.9	11.2	1.7	.0000	1049.9	1.0	.3409
6	1.75	11.2	.0	11.2	.0078	3149.7	2.2	.1554

```
**********************************************************************
CASE C ( 12 BRANCH SHOEBOX COOLING RACK)
3500.0
0.01
60.0
8
12
3.75
64.
8,2, 1.0,1.0,1.0
2,3, 1.0,1.0,2.0
3,4, 1.0,1.0,0.5
4,8, 1.0,1.0,0.6
5,8, 1.0,1.0,0.6
2,6, 1.0,1.0,5.0
7,3, 1.0,1.0,1.0
1,4, 1.0,1.0,2.0
6,5, 1.0,1.0,0.5
6,7, 1.0,1.0,0.6
7,1, 1.0,1.0,0.6
```

```
5,1, 1.0,1.0,5.0
1
6.0
1
OUTPUT CASE C
```

BR	Q	PF	PT	HEAD	ERR	Re	TBF	Y
1	3.48	.0	24.6	-24.6	.0341	6273.0	3.4	.0990
2	1.76	24.6	14.5	10.1	.0393	3168.5	2.0	.1681
3	1.33	14.5	9.6	4.9	.0084	2394.5	1.3	.2685
4	1.74	9.6	.0	9.6	.0371	3138.2	1.9	.1750
5	1.74	9.6	.0	9.6	.0366	3134.8	1.9	.1752
6	1.72	24.6	14.4	10.2	.0386	3104.5	2.1	.1635
7	-.43	13.2	14.5	-1.3	.0000	773.9	1.0	.3409
8	.41	10.8	9.6	1.2	.0000	743.6	1.0	.3409
9	1.32	14.4	9.6	4.8	.0072	2381.3	1.3	.2710
10	.40	14.4	13.2	1.2	.0000	723.2	1.0	.3409
11	.83	13.2	10.8	2.4	.0000	1497.1	1.0	.3409
12	-.42	9.6	10.8	-1.2	.0000	753.5	1.0	.3409

Appendix C: Computer Code

```
$debug
$LIST
c  program for CALCULATING PRESSURES AND FLOW RATES IN PIPILINE NETWORKS
C ******************************************************************
C  PROGRAM flow.FOR WRITTEN BY PROF. RON J PIEPER
C                              NPS  408 6562101
C                              AUG. 30, 1994
C                     SEE FILE flowINP.DAT FOR INPUT
C                     SEE FILE flowOUT.DAT FOR OUTPUT
C                     SEE FILE flowDIA.DAT FOR DIAGNOSTICS ON INPUT
C                     DESIGNED TO HANDLE UP TO 40 BRANCHES
C                              CAN BE ADJUSTED BY INCREASING DIMENSION
C                              OF THE ARRAYS
C *******************************************************************
c  mu                   fluid viscosity (lb-sec/ft^2)
c  rho                  fluid density  (lb/ft^3)
c  g                  acceleration due to gravity (ft/sec^2)
c  nu                 specific gravity rhoxg
c  pi                 pi
c  ren#               reynold's #  (rho*vel*d/mu)
c  f                  friction factor
c  vel                velocity of fluid
c  pres               pressure of the  fluid
c  ****************pipe variables ************************************
c  ell                pipe length(s)
c  d                  pipe diameter(s)
c  e                  roughness
c  rat                roughness ratio
C HCLF                HEADLOSS COUPLING FACTOR
c **********  F FACTOR  *********************************
C  TURB               TURBULENCE FACTOR = F*2100/64 (=1.0 LAMINAR REG)
C  MAXERR             MAXERR IN TURBULENCE FACTOR
c                     FOR REFERENCE ALSO SEE ITERATIVE COOLBROOKE-WHITE
C                     AND ALSO SEE DIRECT CHURCHILL eQ ( IN PLOT PROGRAM)
c                     applies for ren# > REYEDGE
C REDGE             THE LEFT EDGE OF TRANSITION REGION
C                     REYNOLDS# WHERE COOLBROOK BEGINS
c ***************************************************************
c ******  network constants ************************************
c N                   the numver of junctions in the system
```

```
c N1                 the # junction - datum node (n-1)
c B                  the # branches in the system
C S                  the # of sources
c ***************************************************************
c             ASSUMPTIONS
C             1.   COOLBROOKE-WHITE FORMULAE FOR REY#> redge
C             2.   WORK IN ENGLISH UNITS
C             3.   FLUID IS INCOMPRESSIBLE
c             4.   ALL PIPES ARE CIRCULAR IN SHAPE
c             5.   THE CORRECT SOLUTION IS OBTAINED BY ITERATION
C ****************************************************************
      integer N,N1,B,QB,S,DN
      integer NT(40), NF(40),X, itest
      real mu, rho, g, pi, C(40,40), d(40), ell(40), r(40),QS1,QS(40,1)
      REAL RLAM(40),TURB(40),Ct(40,40),Cy(40,40),YLAM(40), Y(40,40)
      REAL IS(40,1),YN(40,40),P(40,1), YH(40,1),V(40,1), REY(40,1)
      REAL AREA(40),ISI(40,1),YNI(40,40),Q(40,1),ERR(40),ep(40),rat(40)
      REAL TURBF,MAXERR,QG(40),FCROS(40)
      REAL REDGE, EPS,H(40,1),HCLF
      CHARACTER*20  FORM
      CHARACTER*3 BR
      CHARACTER*7 NAME(11)
      BR='BR '
      NAME(7)='   TBF '
      NAME(4)='  HEAD'
      NAME(2)='  PF '
      NAME(3)='  PT '
      NAME(6)='  Re  '
      NAME(1)='   Q'
      NAME(8)='  Y  '
      NAME(5)='  ERR'

4     FORMAT (A12)
      OPEN (UNIT=7,FILE='flowDIA.DAT',STATUS='NEW',ACCESS='SEQUENTIAL')
      OPEN (UNIT=8,FILE='flowINP.DAT', STATUS='OLD',ACCESS='SEQUENTIAL')
      OPEN (UNIT=9,FILE='flowOUT.DAT', STATUS='NEW',ACCESS='SEQUENTIAL')
*****************************************************************
      g=32.174
      pi=3.1415926
      READ(8,8)
8     FORMAT (/,/,/,/,/,/,/,/,/,/,/)
C     write(*,13)
      write(7,13)
13    format ('INPUT THE REYNOLDS # FOR RIGHT EDGE TRANSITION ')
```

```
      read(8,30) redge
      WRITE(7,30) REDGE
C     write(*,23)
      write(7,23)
23    format (' enter the maximum error IN TURBULENCE FACTOR desired' )
      read(8,30) MAXERR
      WRITE(7,30) MAXERR
C     WRITE(*,21)
 21   FORMAT ('ENTER THE AVERAGE HEADLOSS COUPLING FACTOR,TYPICAL #60')
      READ(8,30) HLCF
      write(7,21)
      write(7,30) HLCF
C     write(*,10)
      write(7,10)
10    format (' enter the number of nodes in the rack system N>100 ' )
      read(8,15) N
      N1=N-1
15    format(I5)
      write(7,15) N
C     write(*,15) N

C     write(*,20)
      write(7,20)
20    format (' enter the number of branches in the rack system B>N ' )
      read(8,15) B
C     write(*,15) B
      write(7,15) B
****************************************************************************
C     write(*,25)
      write(7,25)
25    format (' input the viscosity __  X 10^-5 lb-SEC/ft^2 ' )
      read(8,30)  mu
      WRITE(7,30) mu
C     WRITE(*,30) mu
      mu=mu*.00001
30    format (f10.4)
C     write(*,35)
      write(7,35)
35    format( ' input the specific weight  lb/ft^3' )
      read(8,30) sw
      rho=sw/g
C     write(*,30) sw
      write(7,30) sw
```

```
c                   Node-branch matrix
c
c    matrix initialization
c
      do 40 i=1,N1
        do 40 j=1,B
40    C(i,j)=0
      do 42 i=1,B
      NF(i)=0
42    NT(i)=0
C
C
C     receive dat from the keybord to develop a,d and ell matrices           D
c     for circular pasages

c     start loop on branches to input node to node information
      do 50 i=1,B
      write(7,55) i
55    format (' at branch  number ', 2x, i5)
C     write(*,60)
      WRLTE(7,60)
60    format('ENTER nf(i),nt(i),length(ft),diameter(in),ROUGHNESS-mil')
      read(8,*) NF(i),NT(i),ell(i),d(i),ep(i)
      ep(i)=ep(i)*.001
      rat(i)=ep(i)/d(i)
c   convert from mils to inches
      WRITE(7,*) NF(i),NT(i),ell(i),d(i),ep(i),rat(i)
C     WRITE(*,*) NF(i),NT(i),ell(i),d(i),ep(i),rat(i)
C 70    format( i5,i5,f8.3,f8.3,f9.5)
c *************  calculate the coolbrook crossing value at R= 3500
c         colebrook-white formula page 198 of fox, let f goto 4f
            sqf=(64.0/2100.0)**.5
 3100        save=sqf
            sqf=1.0/(-2.0*log10(2.51/(REDGE*save) + rat(i)/3.7))
            IF (SAVE .EQ. SQF) GOTO 4000
            error=abs(save-sqf)/sqf
            if (error .gt. .001) goto 3100
 4000         fcros(I)=sqf**2
C     WRITE(*,*) 'I,FCROS(I),RAT(I)', I, FCROS(I), RAT(I)
50    continue
C  SECTION ON SOURCE INPUT
c        INPUT SOURCES, S OF THEM
C        QS  MATRIX OF FLOW SOURCES
C        QB  SOURCE BRANCH
```

```
C         QS1 SOURCE STRENGTH
C         WRITE(*,90)
          WRITE(7,90)
90     FORMAT( ' INPUT THE NUMBER OF SOURCES ')
          READ(8,92) S
          write(7,92) S
C         write(*,92) S

 92    FORMAT(I5)
       DO 98 I=1,S
C      WRITE(*,95) I
       WRITE(7,95) I
 95    FORMAT( 'INPUT THE SOURCE STRENGTH FOR THE', I5,'TH SOURCE')
       READ(8,30) QS1
       write(7,30) QS1
C      write(*,30) QS1

       QS1=QS1/450
C     CONVERT FROM GAL/MIN TO  FT^3/SEC  CONVERSION FACTOR 1/450
C     X GAL/MIN= (450)^-1 FT^3/SEC
      . WRITE(7,93)
C       WRITE(*,93)
93     FORMAT(' INPUT THE BRANCH NUMBER OF THE SOURCE ')
       READ(8,15) QB
       write(7,15) QB
C      write(*,15) QB
98     CONTINUE
       DO 100 K=1,B
       IF (K .EQ. QB) THEN
             QS(K,1)= QS1
              ELSE
             QS(K,1)=0
          ENDIF
100    CONTINUE
c  Start setting up ' C ' matrix
       do 120 i=1,B
         if (NF(i) .LT. N) then
               C(NF(i),i)=1
         endif
           if( NT(i) .LT. N) then
                  C(NT(i),i)=-1
         endif
c      convert diameter in inches to feet
        d(i)=d(i)/12.0
```

```
      area(i)=pi*(d(i)**2)/4
       length=ell(i)
c     60 diameters accounts for in/out head loss
       addl=HLCF*d(i)
       ell(i)=length+ addl
C   CALCULATE THE LAMINAR RESISTANCES FOR EACH BRANCH
       RLAM(i)= 32.0* mu* ell(i)/(AREA(I) * (d(i))**2)
       YLAM(I)=1/RLAM(I)
 120  CONTINUE
C  WRITE "C" MATRIX TO SCREEN AND RECORD FOR CHECK
       WRITE(7,123)
C      WRITE(*,123)

 123   FORMAT( ' THE  C MATRIX  BELOW, TRANSPOSED  ' )
       DO 126 I=1,B
C      WRITE(*,125) (C(J,I),J=1,N1)
       WRITE(7,125) (C(J,I),J=1,N1)

 125   FORMAT(8(1X,F8.4))
 126   CONTINUE
C  WRITE THE LAMINAR RESISTANCE AND ADMITTANCE VECTORS
       WRITE(7,*) 'THE LAMINAR RESISTANCE AND ADMITTANCE VECTORS'
C      WRITE(*,*) 'THE LAMINAR RESISTANCE AND ADMITTANCE VECTORS'
       DO 130 I=1,B
       WRITE(7,*) RLAM(I),YLAM(I)
C      WRITE(*,*) RLAM(I),YLAM(I)
       TURB(I)=1.0
 130   CONTINUE
C            MATRIX MULTIPLICATION OF MATRICES
C
C
      DO 140 J=1,B
      DO 140 K=1,B
      Y(J,K)=C
      IF (J .EQ. K) THEN
          Y(J,J) = YLAM(J)
      ENDIF
140   CONTINUE
C  MATRIX MANIPULATION SECTION
c  N1 rows of C
c  b  columns of C
c  CT transpose of C
      CALL TRANSP(C,CT,N1,B)
C    write(*,*) ' write transpose of matrix C '
```

```
      write(7,*) ' write transpose of matrix C '
      call Warray(CT,B,N1)

C    write(*,*) ' write matrix Y'
      write(7,*) ' write matrix Y'
      call Warray(Y,B,B)


C  PROPOSED ENTRY POINT FOR ITERATIVE SCHEME
250    call matmul(Cy,C,Y,N1,B,B)
C     write(*,*) ' write PRODUCT C*Y   N1XB '
      write(7,*) ' write PRODUCT C*Y   N1XB '
      call Warray(Cy,N1,B)

      CALL MATMUL(YN,Cy,Ct,N1,B,N1)
c  node flow source vector CQs
      call matmul( Is,C,QS,N1,B,1)
c   commented out  artifact of old scheme
       DO 200 I=1,N1
       IS(I,1)=-IS(I,1)
       ISI(I,1)=IS(I,1)
 200   CONTINUE
      DO 295 I=1,N1
      DO 295 J=1,N1
        YNI(I,J)=YN(I,J)
 295   CONTINUE
C  SECTION  ALSO WRITES THE ARRAY PRIOR TO BEING cHOLESKY PROCESSED
       WRITE(7,*) 'YNI MATRIX PRIOR TO CHOLESKY  '
C      WRITE(*,*) 'YNI MATRIX PRIOR TO CHOLESKY  '
       call Warray(YNI,N1,N1)
       WRITE(7,*) 'ISI MATRIX PRIOR TO CHOLESKY  '
C      WRITE(*,*) 'ISI MATRIX PRIOR TO CHOLESKY  '
       call Warray(ISI,N1,1)
       call CHOLESKY(YNI,ISI,N1)
       WRITE(7,*) 'YNI MATRIX FOLLOWING CHOLESKY  '
C      WRITE(*,*) 'YNI MATRIX FOLLOWING CHOLESKY  '
       call Warray(YNI,N1,N1)
C  P  NODE MATRIX  P=YN^-1*IS
C  H  BRANCH PRESSURE CT*P
C  Q  BRANCH FLOW RATE
C YH  MATRIX PRODUCT Y*H
       DO 296 I=1,N1
       P(I,1)=ISI(I,1)
 296   CONTINUE
```

```
      CALL MATMUL(H,CT,P,B,N1,1)
      CALL MATMUL(YH,Y,H,B,B,1)
      DO 300 I=1,B
      Q(I,1)=YH(I,1)+QS(I,1)
C   DIVIDE PRESSURES IN LB/FT^2 BY 144 TO CONVERT TO PSI
 300  CONTINUE

c   REYNOLDS NUMBER REGIME DETERMINATION
     DO 5000 I=1,B
     V(I,1)=ABS( Q(I,1)/AREA(I))
     REY(I,1)=rho*V(I,1)*d(I)/mu
      REN=REY(I,1)
C    WRITE(*,*) ' REYNOLDS #', REY(I,1)
     WRITE(7,*) ' REYNOLDS #', REY(I,1)
     if (REY(I,1) .LE. 2100.0) THEN
      Y(I,I)= YLAM(I)
       TURB(I)=1.0
       TURBAV=1.0
       ERR(I)=0
       F=64.0/2100.0
     else
     if (REY(I,1) .lE. REDGE ) THEN
      X1=REDGE-2100.0
      Y1=Fcros(i) - 64.0/2100.0
      F= 64.0/2100.0 + Y1*SIN(PI* (REN-2100)/(2*X1))
      else
c        colebrook-white formula
          sqf=(64.0/2100.0)**.5
1310         save=sqf
          sqf=1.0/(-2.0*log10(2.51/(Ren*save) + rat(i)/3.7))
C         IF (SAVE .EQ. SQF) GOTO 1400
           error=abs(save-sqf)/sqf
           if (error .gt. .001) goto 1310
1400         f=sqf**2
      endif
      TURBF=F*Ren/64.0
      TURBAV=(TURB(I)+TURBF)/2.0
      Y(I,I)=1/(RLAM(I)*TURBAV)
      ERR(I)=ABS(TURBAV-TURB(I))/turb(i)
      TURB(I)=TURBAV
      ENDIF
C    WRITE(*,*) I,'FRICTION FACTOR', F,'THE TURB FACTOR',TURBAV
     WRITE(7,*) I,'FRICTION FACTOR', F,'THE TURB FACTOR',TURBAV
5000  CONTINUE
```

```
      EPS=0
      DO 5100 J=1,B
C     WRITE(*,*) ' THE ERROR FOR BRANCH',J,' IS',ERR(J)
      WRITE(7,*) ' THE ERROR FOR BRANCH',J,' IS',ERR(J)
      IF (ERR(J) .GT. EPS ) EPS=ERR(J)
5100  CONTINUE
      IF (EPS .GT. MAXERR) GOTO 250
C C CCCCCCC  NOTE ERR DEFINED IN TERMS OF TURBF IS ALREADY NORMALIZED
        WRITE(9,5900) BR,NAME(1),NAME(2),NAME(3),NAME(4),
     + NAME(5),NAME(6),NAME(7),NAME(8)
5900  FORMAT(A3,8(1X,A7))
C
C    READY TO BEGIN OUTPUT OF SOLUTION
C      WRITE(*,*) 'BRANCH,FLOW(Q),PF(MPSI),PT(MPSI), head, ERR,
C    + REY#, TURB FACTOR,  FLOW(Q-QS), '
       WRITE(7,*) 'BRANCH,FLOW(Q),  presURE F,  pres T, head, ERR,
     + REY#, TURB FACTOR, FLOW(Q-Qs)'
       DO 7000 J=1,B
c     Sources calculated in terms of effective pressure diff ( thevenin)
C  NOTE THE FLOW VALUES ARE IN GAL/MIN
C    CONVERT FLOW VALUES TO GAL/MIN
C   DIVIDE PRESSURES IN LB/FT^2 BY 144 TO CONVERT TO PSI
C   ADMITTANCE Y CONVERTED TO (GAL/MIN)/mPSI
c  * 1000 to convert to mpsi
       PF=P(NF(J),1)/144.0*10**3
       PT=P(NT(J),1)/144.0*10**3
       H(j,1)=H(j,1)/144.0*10**3
       Y(J,J)=Y(J,J)*450.0*144.0/10**3
       QS(J,1)=450.0*QS(J,1)
       Q(J,1)=Q(J,1)*450
C      QG(J)=Q(J,1)-Qs(J,1)
C      WRITE(*,6000)J,Q(J,1),PF,PT,H(j,1),ERR(j),
C    + REY(J,1),TURB(J),Y(J,J)
       WRITE(7,6000)J,Q(J,1),PF,PT,H(j,1),ERR(j),
     + REY(J,1),TURB(J),Y(J,J)
       WRITE(9,6000)J,Q(J,1),PF,PT,H(j,1),ERR(j),
     + REY(J,1),TURB(J),Y(J,J)
6000  FORMAT(I3,1X,F7.2,3(1X,F7.1),1X,F7.4,2(1X,F7.1),1X,F7.4)
7000  CONTINUE
      WRITE(*,*) ' *******************************************'
      WRITE(*,*) '                                            '
      WRITE(*,*) ' SEE FILE flowOUT.DAT FOR OUTPUT            '
      WRITE(*,*) ' SEE FILE flowDIA.DAT FOR CHECKING INPUTS     '
      WRITE(*,*) '                                            '
```

```
      WRITE(*,*) ' ******************************************'
      STOP
      END

      subroutine cholesky(a,b,n)
C  SOLVES THE PROBLEM AX=B
C     X UNKNOWN N TUPLE
C     A KNOWN RANK 2 MATRIX OF ORDER N
C     B KNOWN N TUPLE ( UPON ENTRY )
C     B=X ( UPON EXIT)
      real b(40,1)
      real a(40,40)
      write(7,*) ' in cholesky routine, matrix order n follows '
      call Warray(a,n,n)
      call decomp(a,n)
      b(1,1)=b(1,1)/a(1,1)
      do 10 i=2,n
      d=b(i,1)
      il=i-1
      do 5 l=1,il
5     d=d-a(l,i)*b(l,1)
10    b(i,1)=d/a(i,i)
      b(n,1)=b(n,1)/a(n,n)
      n1=n-1
      do 30 l=1,n1
      k=n-l
      k1=k+1
      do 20 j=k1,n
20    b(k,1)=b(k,1)-a(k,j)*b(j,1)
30    b(k,1)=b(k,1)/a(k,k)
      return
      end

      subroutine matmul(c,a,b,n,m,l)
C     SOLVES THE PROBLEM C=A*B
c     C UNKNOWN MATRIX  lxn
c     A KNOWN MATRIX MxN
c     b KNOWN MATRIX  lXm
      real a(40,40), b(40,40), c(40,40)
C  n     # columns of a and c
c  m     # rows of a and columns of b
c  l     # rows of b and c
      do 25 i=1,n
      do 25 j=1,l
```

```
          c(i,j)=0
          do 25 k=1,m
25            c(i,j)= c(i,j)+ a(i,k)*b(k,j)
      return
      end

      subroutine decomp(a,n)
      real a(40,40)
      i=1
      write(7,*) ' now in decomp routine  '
      write(7,*) 'the order of matrix a is', n
      do 410 ii=1,n
      write(7,35) (a(ii,jj),jj=1,n)
410   continue
      if ( a(1,1) ) 1, 1, 3
 1    write(7,2)
      write(7,*) i,a(i,i)
 2    format( ' zero or negative radicand ')
      goto 200
 3    a(1,1)=sqrt(a(1,1))
      do 10 j=2,n
 10   a(1,j)=a(1,j)/a(1,1)
      do 40 i=2,n
      i1=i-1
      d=a(i,i)
      do 20 l=1,i1
      dhold=d
 20   d=d-a(l,i)*a(l,i)
      if ( d .eq. 0)  then
                  WRITE(7,*) ' d is zero   '
                  write(7,*) i,a(l,i), dhold
      endif
      write(7,*) ' i, matrix(i,i) follow '
      write(7,*) i, a(i,i)
      if (a(i,i)) 1,1,21
 21   a(i,i)=sqrt(d)
         if ( i .eq. n) goto 45
         i2=i+1
      do 40 j=i2,n
      d=a(i,j)
      do 30 l=1,i1
30    d=d-a(l,i)*a(l,j)
40    a(i,j)=d/a(i,i)
45    do 50 i=2,n
```

```
c      zero out lower part of matrix
       i1=i-1
       do 50 j=1,i1
50     a(i,j)=0
       write(7,*) 'made it through decomp , matrix follows '
       do 400 ii=1,n
       write(7,35) (a(ii,jj),jj=1,n)
       format(8(2x,f8.4))
400    continue
200    return
       end
       SUBROUTINE TRANSP(A,AT,NR,NC)
       REAL A(40,40), AT(40,40)
       DO 20 J=1,NC
       DO 20 K=1,NR
20     AT(J,K)=A(K,J)
       RETURN
       END

       subroutine warray(a,n,m)
c  writes array to screen and to diagnostic file
       real a(40,40)
c    n rows of  a
c    m columns of a
       do 20 j=1,n
       write(7,30) (a(j,i),i=1,m)
C      write(*,30) (a(j,i),i=1,m)
20     continue
30     format (8(1x,f8.4))
       return
       end
```

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